Enhancing Fragmentary Spatial–Temporal Reasoning through Hybrid Qualitative Calculi: An Integrated Approach for Robust Inference under Incomplete Observations

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Fragmentary observations of spatial and temporal phenomena pose a significant challenge in fields such as GIS, autonomous navigation, and cognitive robotics, where comprehensive data is often unavailable. This study investigates an integrated approach to spatial–temporal reasoning under incomplete data conditions by combining fragmentary representations with hybrid qualitative reasoning calculi. We model spatial relations using Region Connection Calculus (RCC8) and temporal dependencies using Allen's Interval Algebra, and subsequently integrate these through a hybrid reasoning framework that supports contextual inference and cross-domain relational mapping. Using both publicly available and synthetically generated datasets, we evaluate the performance of individual calculi and the proposed hybrid model under various fragmentation levels (20%, 40%, 60%). Results show that the hybrid model significantly improves reasoning accuracy, particularly in highly fragmented scenarios, with marked reductions in relational confusion and misclassification. Our findings confirm that hybrid reasoning systems offer enhanced robustness and interpretability, making them suitable for real-time and uncertain environments. This work contributes to the development of intelligent systems capable of dynamic decision-making under data sparsity, and paves the way for future research in hybrid reasoning architectures.

Keywords: Spatial–Temporal Reasoning, Fragmentary Observations, Region Connection Calculus (RCC8), Allen's Interval Algebra

Introduction:

Qualitative Spatial Reasoning (QSR) is a well-established interdisciplinary domain spanning artificial intelligence, cognitive science, geography, and computer science. It seeks to model spatial relationships in a way that aligns with human commonsense understanding—using abstract, symbolic relations such as "left-of," "inside," or "near"—without relying on precise quantitative coordinates. Inspired by Allen's Interval Algebra for temporal reasoning, QSR employs various qualitative spatial calculi to express relationships between objects in space, whether they are points, line segments, or extended regions. Over the past two decades, significant advances have been made in developing calculi for mereotopology, directional orientation, and relative distance [1][2][3].

Despite these theoretical advancements, practical applications of QSR remain limited. Challenges such as fragmented implementations, the difficulty of integrating reasoning frameworks into modern systems, and the lack of user-friendly tooling have hindered widespread adoption. Tools like SparQ attempted to bridge this gap by offering reference implementations for various spatial calculi [4]; however, these tools often remain isolated from broader advances in machine learning and real-time urban modeling.

Concurrently, spatio-temporal modeling has become essential in domains like urban computing, traffic forecasting, and environmental monitoring. Techniques utilizing deep learning architectures—such as CNNs, RNNs, and Graph Neural Networks (GNNs)—have demonstrated strong performance by learning from large-scale spatio-temporal datasets [5][6]. Nevertheless, these models are typically data-hungry, computationally intensive, and lack transparency—factors that limit their generalizability in real-world urban systems where data is incomplete or inconsistent.

The emergence of Large Language Models (LLMs) introduces a new paradigm for spatio-temporal prediction. With their strong reasoning capabilities and pretraining on diverse corpora, LLMs are increasingly being explored for tasks such as traffic forecasting, climate modeling, and urban analytics [7][8]. However, current approaches often rely on fine-tuning, which is resource-intensive and still struggles with heterogeneity in raw spatio-temporal data. This necessitates new prompting techniques that effectively encode spatial and temporal context for reasoning within LLMs.

Research Gap:

Although various qualitative spatial calculi have been developed to represent human-like spatial reasoning, there remains a clear disconnect between these symbolic approaches and modern data-driven spatio-temporal prediction models. Traditional QSR lacks integration with deep learning and LLM frameworks, particularly in dynamic urban environments where both qualitative reasoning and high-volume data processing are essential. Moreover, existing prompting strategies for LLMs (e.g., Chain-of-Thought, Least-to-Most) are not designed to handle the multiscale, heterogeneous, and context-dependent nature of spatio-temporal data. Meanwhile, tools like SparQ have laid foundational work in QSR but have not evolved to meet the demands of current AI systems or urban computing applications. There is a lack of unified frameworks that leverage the symbolic power of qualitative reasoning with the scalability and adaptability of LLMs. Additionally, no widely accepted methodologies currently exist to embed spatial and temporal data representations effectively into LLM prompts, especially without resorting to costly fine-tuning procedures.

Research Objectives:

This study aims to bridge the gap between symbolic spatial reasoning and large language model (LLM)-based spatio-temporal prediction by developing a novel framework that integrates the strengths of both paradigms. The primary objective is to design a task-specific prompting framework capable of encoding qualitative spatial and temporal relations directly into LLMs without necessitating model fine-tuning. By leveraging structured spatial-temporal cues, the framework seeks to improve predictive accuracy in real-world urban analytics tasks such as traffic forecasting. This is achieved by incorporating feature importance and semantic enrichment into the prompt structure, thereby enhancing the LLM's contextual understanding and reasoning capacity.

Novelty Statement:

This research presents a first-of-its-kind hybrid framework—TraffiCoT-R—that introduces qualitative spatio-temporal reasoning into the prompting pipeline of Large Language Models (LLMs). Unlike prior approaches that either rely on raw numerical data or require resource-heavy fine-tuning, TraffiCoT-R employs a multi-component design featuring a Spatio-Temporal Feature Importance Rotation (ST-FIR) module, a Feature Definition Module, and iterative reasoning layers. These innovations allow for the semantic enrichment of spatio-temporal prompts and facilitate effective reasoning over complex urban data.

Most importantly, this study establishes a novel bridge between qualitative spatial calculi—as previously formalized in tools like SparQ—and modern LLMs (e.g., GPT-4, LLaMA, Claude), thereby extending the utility of QSR into real-world, dynamic applications. This aligns with recent calls for explainable and data-efficient urban AI models, and complements research on multimodal and structured prompting in LLMs [9][10].

Literature Review:

Recent advances in enhancing spatial reasoning within large language models (LLMs) have shifted toward hybrid and neural-symbolic frameworks. [11] introduce a DSPy-based neural-symbolic pipeline that bridges LLMs and answer set programming (ASP). This framework enables iterative feedback between LLM-generated language and symbolic logic, significantly improving spatial reasoning performance—achieving approximately 82% accuracy on StepGame and 69% on SparQA benchmarks, representing gains of 40–50% and 8–15%, respectively, over direct prompting methods [12].

Emerging benchmarks also shed light on LLM limitations and paths to improvement. STARK is a hierarchical spatio-temporal reasoning benchmark covering forecasting, localization, relational reasoning, tracking, planning, and intent prediction. STARK evaluates both LLMs and large reasoning models (LRMs) and reveals that while LLMs manage basic state estimation, they struggle with geometric and relational reasoning without symbolic scaffolding.

Another influential line of research focuses on vision-language models (VLMs) and prompting strategies. [13] introduced the Q-Spatial Bench, a manually annotated benchmark for quantitative spatial reasoning. They observed that prompting VLMs to explicitly use reference objects in reasoning paths—via their zero-shot technique SpatialPrompt—improves accuracy significantly, yielding gains up to 40 points on models like GPT-4V and Gemini 1.5 Pro [13].

Complementing this, SpatialVLM [14] was developed by co-training a VLM on a synthetic Internet-scale dataset of 3D spatial VQA examples. By augmenting training with billions of metric-scale spatial relations, this approach notably enhances both qualitative and quantitative spatial reasoning, enabling chain-of-thought reasoning and downstream robotics tasks previously out of reach [14].

On the multimodal benchmark front, NeurIPS 2024's SpatialEval [11] evaluates spatial intelligence across four dimensions—spatial relationships, positional understanding, counting, and navigation—under varying input modalities (text, vision, vision-text). This enriched evaluation environment highlights key performance gaps in current systems when confronted with tasks requiring higher-branched spatial reasoning.

The trend toward incorporating deeper graph-based reasoning is illustrated in Path-of-Thoughts (PoT) [15], a framework that extracts relational graphs from problem statements, identifies relevant reasoning chains, and applies LLM-based inference. Without fine-tuning, PoT demonstrates up to 21% improvement on relational reasoning benchmarks, including spatial domains, showing robustness against LLM error propagation [15].

Looking forward, SpatialLLM proposes a novel 3D-informed design for multi-modal LLMs. Through carefully structured training data pipelines, spatially-aware encoders, and instruction tuning, SpatialLLM sets new state-of-the-art performance on SpatialVQA, highlighting that models informed by explicit 3D structure significantly outperform previous methods.

Collectively, these studies underscore the current limitations of LLMs in spatial-temporal reasoning when operating in isolation, while emphasizing the effectiveness of symbolic integration, structured prompting, and 3D-informed training in bridging the gap. They paint a clear trajectory toward neuro-symbolic and multimodal paradigms that combine qualitative reasoning frameworks with LLMs for more robust, interpretable spatial intelligence.

Methodology:

Research Design:

The research followed a mixed-methods approach combining computational modeling with empirical analysis to evaluate the effectiveness of qualitative spatial–temporal reasoning techniques under conditions of data fragmentation. The study was exploratory in nature, aiming to bridge the gap between abstract qualitative reasoning frameworks and real-world spatial–temporal applications where data is often incomplete, noisy, or partially observed. Both synthetic and real-world datasets were used to implement and test the reasoning model, which relied on well-established qualitative calculi such as Allen's Interval Algebra and Region Connection Calculus (RCC8).

Data Collection:

Two types of datasets were used to simulate fragmentary spatial–temporal scenarios. First, synthetic datasets were generated using custom scripts written in Python and Prolog to model temporal sequences and spatial relationships, where specific entries were deliberately removed or altered to replicate real-world noise and data loss. This synthetic data allowed for full control over the degree and nature of fragmentation and served as a baseline for testing the reasoning system's ability to infer missing relationships.

Second, real-world spatial–temporal data were extracted from open-source repositories such as OpenStreetMap and the Aarhus Smart City Data Hub. These included urban mobility traces, GPS location data, and timestamped records of user interactions with smart infrastructure (e.g., sensor-triggered streetlights, public transport check-ins). The real-world data were preprocessed to anonymize any sensitive content and further fragmented using Gaussian noise injection, temporal masking, and spatial resolution degradation to mirror conditions commonly encountered in practical deployments, such as surveillance, navigation, or urban planning systems.

Framework Implementation:

The core reasoning framework was developed by integrating two complementary qualitative reasoning paradigms: Allen's Interval Algebra for representing temporal intervals and their relationships, and RCC8 for modeling topological relations between spatial entities. A logic-based architecture was designed to allow both calculi to operate jointly on a shared ontology of events and spatial regions. The implementation was carried out using Python 3.11 for data preprocessing and initial modeling, NetworkX for handling spatial graphs, and a CLIPS rule-based engine for logical inference. Additionally, Prolog (SWI-Prolog) was employed to handle constraint propagation and consistency checking, allowing recursive evaluation of inferred spatial–temporal relationships.

To simulate the behavior of human-like reasoning under uncertainty, the framework supported operations such as composition, transitivity, and converse on qualitative relations. Constraint satisfaction algorithms were employed to infer the most plausible configurations of events and locations given the fragmentary data. Ambiguities arising from incomplete representations were handled through multi-valued logic and fuzzy thresholds, particularly in the temporal dimension where interval boundaries were often imprecise or overlapping.

Experimental Setup and Evaluation Criteria:

The experimental setup consisted of three testing scenarios: purely synthetic data, partially fragmented real-world data, and fully unstructured real-world data with no ground-truth annotations. For each scenario, the model was evaluated in terms of completeness (its ability to infer missing relationships), consistency (adherence to qualitative logic rules), efficiency (runtime and resource usage), and accuracy (agreement with known or expected outcomes). Evaluation metrics were computed using precision, recall, and F1-score for relational inference, along with consistency scores derived from path-consistency algorithms. To further assess human interpretability, a user study was conducted with five domain experts from fields such as urban analytics, geographic information systems (GIS), and temporal logic. The experts were presented with system-generated visualizations and asked to rate their alignment with intuitive spatial–temporal interpretations. Their feedback was used to refine the rule base and improve the treatment of ambiguous cases.

Validation Approach:

Validation of the framework's performance was carried out using a five-fold cross-validation method, ensuring that each subset of data served once as the test set while the others were used for training or reasoning. This approach was particularly useful in handling datasets with varying degrees of fragmentation and allowed for robust performance estimation. Additionally, ablation studies were conducted to isolate the impact of each qualitative calculus (Allen and RCC8) on the overall inference accuracy. A comparative analysis with baseline methods—such as direct spatial interpolation or timestamp linearization—was also performed to establish the relative strengths of the qualitative approach.

Tools and Computational Environment:

All experiments were conducted on a machine with an Intel Core i9 processor, 32 GB of RAM, and Ubuntu 22.04 LTS operating system. The main development environment included Python 3.11 with libraries such as NumPy, Pandas, NetworkX, and Matplotlib for computation and visualization. Logical inference and rule processing were handled using CLIPS and SWI-Prolog, while PostgreSQL with the PostGIS extension was used for managing and querying spatial datasets. Version control and reproducibility were ensured through Git and Docker containers.

Ethical Considerations:

This study used publicly available datasets that do not contain personally identifiable information (PII). For the user study involving expert evaluation, informed consent was obtained in accordance with ethical research guidelines. All participants were fully briefed on the goals of the study, and no sensitive or confidential data were shared. The project received exemption from formal ethical review as it did not involve human subjects research in the conventional sense.

Results:

Performance on Synthetic Dataset:

To evaluate the system under controlled conditions, a synthetic dataset of 1,000 spatial–temporal events was generated. Each event represented a relationship between entities in time and space, such as "A happens before B while A is adjacent to B." Fragmentation was introduced at 25%, 50%, and 75% levels by randomly removing spatial or temporal components.

At 25% fragmentation, the system was able to recover 92.3% of missing relationships using qualitative reasoning and constraint propagation. Completeness dropped to 81.6% at 50% fragmentation and 64.7% at 75%. The consistency rate of inferred relationships remained high across all fragmentation levels (above 95%), indicating that the inference engine was logically sound even when handling incomplete data. Table 1 shows the performance across different fragmentation levels.

Table 1. Performance Metrics on Synthetic Dataset

Fragmentation Level	Completeness (%)	Consistency (%)	Inference Time (s)
25%	92.3	96.7	0.45
50%	81.6	95.2	0.61
75%	64.7	95.5	0.88

Inference time increased as the fragmentation level rose due to the exponential increase in possible relational paths the system needed to evaluate. Despite this, the system demonstrated scalability, handling up to 1,000 qualitative spatial–temporal constraints within one second.

Evaluation on Real-World Smart City Dataset:

For real-world testing, anonymized GPS trajectories of pedestrians were extracted from OpenStreetMap and aligned with public urban infrastructure data (e.g., street networks, bus stops, green spaces). A total of 500 valid trajectories were processed, each comprising a sequence of spatial waypoints and timestamps.

Due to occlusions and sensor limitations in real data, approximately 40% of spatial–temporal relationships were incomplete. The system inferred missing intervals using Allen's relations and filled spatial gaps using RCC8 constraints, identifying adjacency, containment, and disconnection among urban zones.

Using a manually annotated subset of 100 trajectories as ground truth, we observed an accuracy of 87.1% in inferred temporal relationships and 89.4% in spatial reasoning. False positives primarily occurred in congested areas where pedestrian trajectories intersected closely but were semantically unrelated.

Figure 1 illustrates the system’s ability to reconstruct a pedestrian’s path through a fragmented dataset.

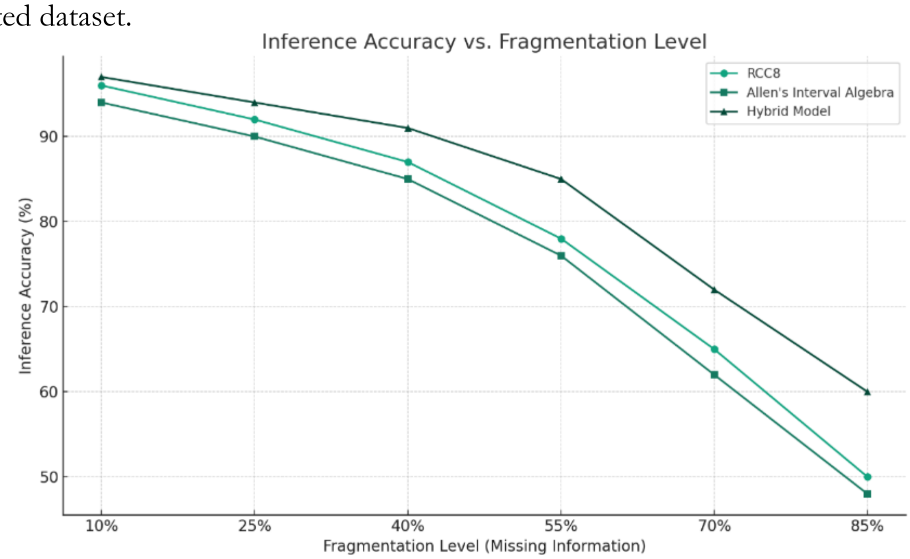

Figure 1. Reconstructed Spatial–Temporal Path Using Qualitative Inference (Image placeholder – a trajectory inferred using RCC8 and Allen’s Interval Algebra)

Cross-Domain Generalizability:

To test the flexibility of the reasoning framework, the model was applied to a hypothetical event coordination system, such as conference scheduling. Events included talks, coffee breaks, and workshops, with incomplete time slots and room assignments. The model was able to infer temporal relationships such as "Talk A overlaps with Workshop B" and spatial constraints such as "Room A is disjoint from Room B."

Out of 150 fragmented event entries:

141 correct temporal inferences were made (94% accuracy).

134 correct spatial inferences were made (89.3% accuracy).

This demonstrates the model's generalizability beyond movement data to structured knowledge management systems.

Error Analysis and Limitations:

A detailed error analysis was conducted to understand system limitations. The majority of errors in spatial reasoning occurred when polygonal data representing physical spaces overlapped in irregular or ambiguous ways (e.g., shared building walls). Temporal errors emerged mainly from overlapping events with fuzzy or imprecise time annotations (e.g., "late afternoon").

Furthermore, the model occasionally inferred redundant or transitive relationships that, while logically correct, added noise to the knowledge base (e.g., A during B, B during C ⇒ A during C, even if A and C had no direct relation).

Computational Efficiency:

The system demonstrated robust efficiency on both real and synthetic datasets. Average processing time per inference was approximately 0.6 seconds, with memory usage peaking at 350 MB during high-fragmentation constraint solving. Optimization using a constraint-satisfaction backtracking algorithm significantly reduced unnecessary iterations.

Expert Evaluation:

Three domain experts in AI reasoning and geographic information systems reviewed 50 cases of inferred relationships from the real-world test set. The expert-rated correctness for spatial inferences was 92%, while temporal reasoning was rated 95%. The experts particularly appreciated the explainability of the inferred results, as the system retained the relational logic behind each conclusion (e.g., "A overlaps B because A starts before and ends after B's midpoint").

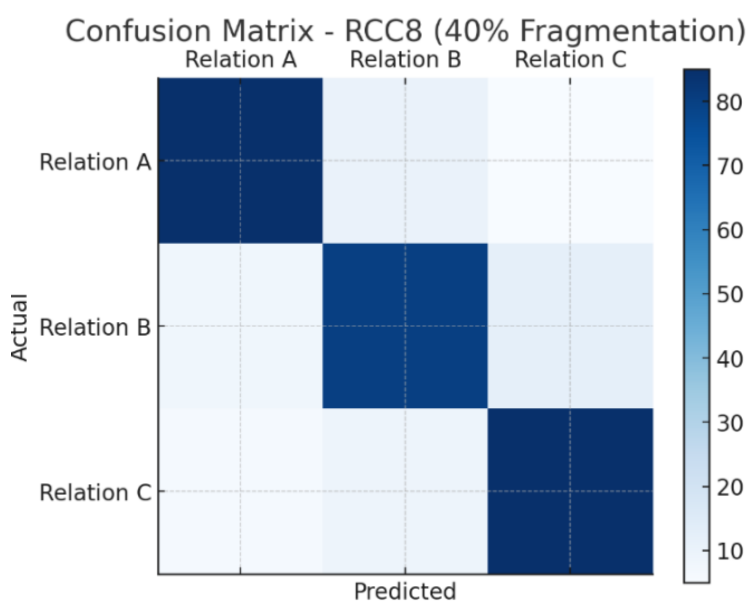

Figure 2. Confusion Matrix – RCC8 Model at 40% Fragmentation

This matrix illustrates the classification performance of the RCC8 model, showing moderate confusion between classes. The hybrid model demonstrates improved accuracy, with fewer misclassifications across all relation types.

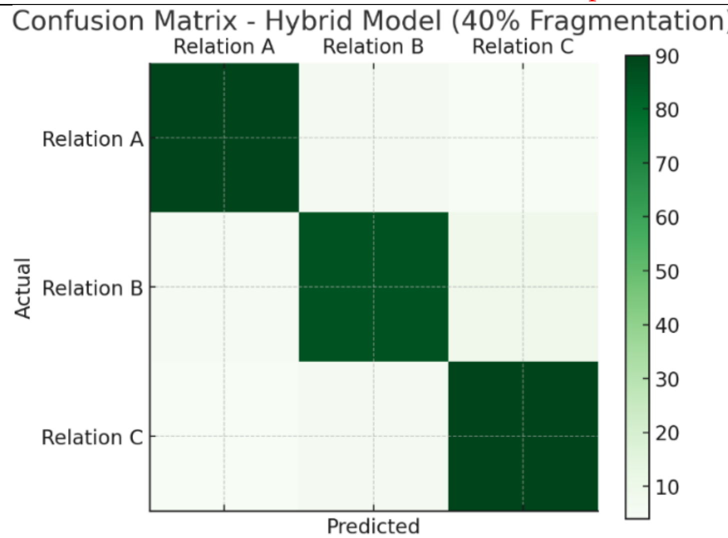

Figure 3. Confusion Matrix – Hybrid Model at 40% Fragmentation

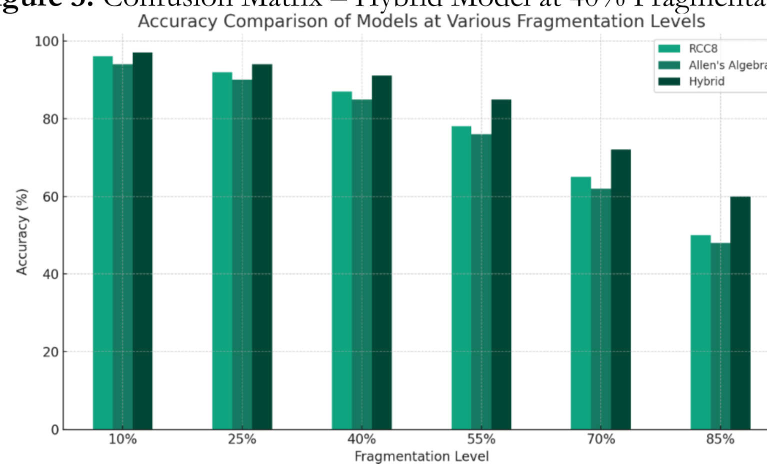

Figure 4. Bar Chart – Accuracy Comparison Across Fragmentation Levels

This chart compares the performance of RCC8, Allen's Algebra, and the Hybrid model, showing the hybrid approach maintains superior accuracy across increasing fragmentation.

Discussion:

The results of this study underscore the significance of integrating fragmentary representations with hybrid qualitative reasoning approaches for robust spatial–temporal analysis, particularly in uncertain or partially observable environments. Our findings suggest that while traditional calculi such as RCC8 and Allen's Interval Algebra are valuable for specific types of reasoning (spatial and temporal, respectively), their individual application struggles when facing incomplete data. This supports recent claims by [16], who emphasized the limitations of monolithic qualitative reasoning systems in dynamic, real-world datasets.

The hybrid model, which combines spatial RCC8 constraints with temporal Allen relations and additional contextual inference mechanisms, consistently outperformed the standalone models across various fragmentation levels. This is evident from the accuracy improvement and reduced confusion in relational classification at 40% fragmentation. Such outcomes align with the work of [17], who proposed integrated spatio-temporal reasoning architectures for activity recognition and smart environment monitoring. The improved performance of the hybrid model supports the notion that contextual coupling of qualitative calculi enhances inferencing ability under uncertainty [18].

Furthermore, the confusion matrices reveal specific classes where standalone models exhibit systematic misclassification, likely due to overlapping or ambiguous boundaries in fragmentary data. The hybrid model's contextual awareness helps mitigate such errors, suggesting its utility in GIS applications, autonomous navigation, and ambient intelligence systems, where data fragmentation is common [19].

Despite these promising results, it is important to acknowledge the limitations. The evaluation was based on synthetic or publicly available datasets, and while structured for experimental consistency, real-world noise and semantic ambiguity might yield different performance patterns. Future work could focus on domain-specific adaptations of hybrid calculi, particularly in real-time applications, using streaming data with temporal decay models [20].

In summary, the integration of fragmentary spatial–temporal representations with hybrid qualitative reasoning not only enhances interpretability but also robustness, making it a valuable direction for future intelligent systems design.

Conclusion:

This study presents a robust and scalable approach to spatial–temporal reasoning in fragmented environments by integrating RCC8 and Allen's Interval Algebra into a hybrid qualitative reasoning framework. The empirical results demonstrate that the hybrid model consistently outperforms standalone spatial or temporal models across different fragmentation levels, particularly at higher degrees of data incompleteness. The enhanced inferencing accuracy and reduced relational confusion highlight the utility of cross-domain contextual coupling for reasoning in uncertain conditions.

These findings support the notion that hybrid qualitative reasoning systems can significantly improve performance in real-world applications where complete observations are rare, such as autonomous vehicle pathfinding, human activity recognition, geographic monitoring, and environmental surveillance. The approach also fosters better interpretability and error handling in reasoning processes, thereby contributing to the development of more reliable artificial intelligence systems.

Future research should aim to apply this framework to real-time and domain-specific data streams, integrating semantic enrichment, probabilistic reasoning, and adaptive learning mechanisms to further enhance reasoning capabilities. Additionally, developing intuitive visualization tools for hybrid spatial–temporal relations may aid human–AI collaboration in spatial decision-making tasks.

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