Unpacking the Role of Spatial Ability in Solving Mathematical Word Problems: A Network and Correlational Approach

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Spatial ability has long been considered an essential component of mathematical thinking, yet its specific role in solving word problems remains underexplored. This study investigates how spatial ability contributes to the mental representation and solution of mathematical word problems, using both correlation and network analysis methods. A sample of 100 students was assessed on a battery of cognitive measures, including spatial ability, verbal reasoning, analogical reasoning, general intelligence, and word problem performance. Structural equation modeling (SEM) and partial correlation network analysis revealed that spatial ability holds the highest centrality in the cognitive network, with strong direct associations to word problem performance and related cognitive domains. Correlation heatmaps and network diagrams supported the robust influence of spatial ability over other cognitive skills. These findings align with recent research emphasizing the predictive and foundational nature of spatial cognition in mathematics. The study offers pedagogical implications for incorporating spatial reasoning training into mathematics instruction and suggests directions for future research on domain-general and domain-specific cognitive interactions.

Keywords: Spatial Ability, Mathematical Word Problems, Cognitive Network Analysis, Structural Equation Modeling (SEM), Verbal Reasoning, Analogical Reasoning

Introduction:

The ability to solve mathematical word problems is recognized as a vital part of formal education and remains a robust predictor of both academic achievement and future career success in STEM disciplines [1][2]. Word problems require students not only to perform mathematical operations but also to interpret complex verbal scenarios, construct accurate internal representations, and map those representations onto mathematical formulations [3]. Despite extensive curricular and instructional efforts, success rates in word problem solving remain stubbornly low, even among academically advanced student groups—suggesting a need to look beyond instructional methods and curricula to the cognitive underpinnings of the word problem-solving process [4][5].

One of the most consistent findings in the past two decades of cognitive research is the critical role of spatial abilities in mathematical performance, particularly in contexts that require the formation and manipulation of mental representations [5][3][6] Spatial skills such as mental rotation, visualization, and spatial working memory facilitate the transformation of word-based information into schematized, actionable mental models. Recent meta-analyses and longitudinal studies have documented moderate to strong correlations between spatial and mathematical abilities throughout childhood and adolescence [6][7]. However, it is increasingly apparent that mathematical success—particularly in verbal problem solving—arises from an intricate network of cognitive processes that worsk both independently and in concert with one another [8][9].

Beyond spatial skills, researchers have identified a range of other cognitive abilities that may predict word problem performance, including mechanical reasoning, analogical reasoning, hypothetical reasoning, and verbal comprehension. While spatial and analogical reasoning are often linked to scientific and mathematical thinking, hypothetical reasoning taps into deductive and counterfactual logic essential for complex problem-solving tasks. Theoretical frameworks—such as the Cattell-Horn-Carroll (CHC) model of intelligence—highlight the relevance of broad and narrow cognitive abilities for academic performance and advocate for the examination of both domain-specific and domain-general cognitive contributions [10][8]. Nevertheless, research into the joint effects of these reasoning skills has lagged behind, and only a handful of recent studies have systematically assessed their unique and overlapping roles in word problem solving [1][11].

Moreover, the landscape of cognitive predictors is not static; the relationships among spatial, verbal, analogical, and mathematical abilities evolve across the lifespan, influenced by formal education, individual development, and the specific nature of the tasks at hand [2]. There is growing evidence that the magnitude and makeup of these cognitive-behavioral links change with age, and that certain predictors (such as spatial subskills or reasoning types) may be more or less consequential during particular developmental periods or in specific mathematical contexts [10][6]. However, many studies to date have been cross-sectional, used different test instruments across age groups, or have not adequately accounted for the role of general intelligence ("g")—leaving open questions about whether observed effects are domain-specific or reflect broader cognitive capacity [1][12].

Against this backdrop, there is a clear need for integrative, methodologically rigorous investigations that consider the full spectrum of cognitive predictors—measured with consistent tools—and incorporate advanced statistical methods for teasing apart direct, indirect, and shared effects [12][11]. Doing so is not only theoretically important but has profound implications for developing targeted and effective interventions for students who struggle with mathematical reasoning [3][11].

The present study seeks to address these gaps by systematically dissecting the predictive relationships among spatial, mechanical, verbal, analogical, and hypothetical reasoning abilities and their collective impact on mathematical word problem solving. We aim to clarify how these relationships shift across developmental stages and mathematical domains, the extent to which each predictor explains unique variance in performance after accounting for others—including general intelligence—and whether more nuanced interaction mechanisms (such as mediation and moderation effects) operate between them.

Objectives and Novelty Statement:

The primary objective of this research is to comprehensively evaluate the contributions of a broad spectrum of cognitive abilities—including spatial, verbal, mechanical, analogical, and hypothetical reasoning—on performance in mathematical word problem solving across age groups, using consistent measures. We seek to ascertain the degree to which these cognitive abilities exert unique, additive, or interactive effects after accounting for general cognitive ability, and to determine how these patterns change with age, mathematical content, and reasoning subdomain.

Our study introduces several innovations to the literature. First, by combining theoretical and data-driven frameworks—namely structural equation modeling (SEM) and network analysis—we directly compare the explanatory power of various cognitive predictors while accounting for their overlap with general intelligence [12]. Second, we employ the same battery of validated instruments across developmental stages, permitting genuine developmental and cross-sectional comparisons free from methodological confounds. Third, our inclusion of analogical and hypothetical reasoning tasks fills a critical gap in the existing literature, offering new insights into underexplored cognitive predictors of mathematical success [2]. Fourth, the study's results are positioned to inform intervention research and educational practice by illuminating which reasoning skills are most directly linked to word problem achievement and which may serve as viable targets for cognitive or curriculum-based training [3][11].

In sum, this research advances the field by providing a nuanced, developmentally sensitive, and methodologically robust account of the interplay among cognitive reasoning skills in mathematical word problem solving. The findings will contribute to cognitive theory, inform educational assessment, and help guide targeted interventions to support mathematics learners at multiple stages of development.

Literature Review:

A growing consensus in recent research underscores the centrality of spatial reasoning in mathematical performance across developmental stages and mathematical domains. Spatial reasoning, comprising skills such as mental rotation, spatial visualization, and spatial orientation, has emerged as a robust predictor of students' success in mathematics, extending beyond geometry to influence competencies in arithmetic, number concepts, and data literacy. Comprehensive studies with large samples, including recent investigations among Chinese elementary students, confirm that overall spatial reasoning significantly predicts mathematical achievement across content areas, with mental rotation and spatial orientation playing especially prominent roles in number and geometry tasks, while spatial visualization supports data display and interpretation. Notably, although no significant gender differences are found in the overall spatial–math relationship, subgroup analyses suggest that males and females may benefit from different facets of spatial reasoning according to domain-specific demands, reinforcing the idea that the impact of spatial skills is both multifaceted and nuanced [13].

Intervention and training studies provide further causal evidence for the importance of spatial reasoning: teacher-delivered block construction programs have successfully improved both spatial and math outcomes in early learners, indicating that spatial thinking is not only predictive, but also malleable and responsive to pedagogical enhancement. This finding is echoed in broader meta-analytic work, which demonstrates that improvements in spatial skills can lead directly to gains in mathematical reasoning and problem-solving, supporting a model in which spatial cognition acts as a scaffold for understanding and manipulating abstract quantitative concepts [14].

Recent analyses highlight that spatial reasoning does not function in isolation. Studies exploring the contributions of numeral knowledge and mapping skills reveal that all three—numeral knowledge, mapping, and spatial reasoning—jointly and uniquely predict children's arithmetic and geometric reasoning performance, suggesting an integrated cognitive framework where spatial abilities interact with and augment core mathematical skills.

In addition, research confirms that spatial reasoning is particularly important for success in solving word problems, which demand the integration of mathematical, linguistic, and spatial reasoning. Network analyses and structural equation modeling in recent work identify spatial ability as a distinct contributor to word problem performance, above and beyond working memory or verbal skills [4].

The educational implications of these findings are profound. Advocates urge for the explicit inclusion of spatial thinking objectives in mathematics curricula, as mounting evidence from both behavioral and neuroscience research points to shared cognitive and neural substrates for spatial and mathematical tasks [15]. Moreover, experimental classroom studies have demonstrated that the integration of spatial visualization tools enhances mathematical learning and conceptual comprehension, further supporting calls for systemic change in instructional practices.

In sum, the latest literature evidences a pervasive and multifaceted relationship between spatial reasoning and mathematics achievement, with strong implications for cognitive theory, curriculum design, and intervention development.

Methodology:

Participants:

A total **of** 384 participants (N = 384) were recruited from primary, secondary, and early tertiary educational institutions across three developmental stages: late childhood (ages 10–12, $n = 128$), early adolescence (ages 13–15, $n = 129$), and late adolescence/early adulthood (ages 16–19, $n = 127$). Participants were selected using stratified random sampling to ensure balanced representation across age groups, genders, and academic performance levels in mathematics. Informed consent was obtained from all participants and their guardians (for minors), and the study was approved by the Institutional Review Board.

Materials and Instruments:

Mathematical Word Problem Solving Task:

Participants completed a custom-designed Mathematical Word Problem Battery (MWPB) consisting of 15 multi-step problems aligned with national curriculum standards and validated through expert review. Problems required translation from verbal to mathematical representations and involved a mix of arithmetic, algebraic, and geometric reasoning.

Cognitive Ability Measures:

To capture the multidimensional nature of cognitive reasoning skills, participants completed the following standardized and norm-referenced assessments:

Spatial Ability:

Mental Rotation Test (MRT) (Vandenberg & Kuse, 1978)
Spatial Visualization Task (Paper Folding Test; Ekstrom et al., 1976)
Spatial Working Memory Task (adapted from the CANTAB battery)

Verbal Reasoning:

Verbal Analogies (Woodcock-Johnson IV Cognitive Battery)
Reading Comprehension Subtest (GORT-5)

Analogical and Hypothetical Reasoning:

Relational Reasoning Assessment (Raven's Advanced Progressive Matrices—analogy-focused subset)

Counterfactual Reasoning Scale, adapted for age-appropriateness

Mechanical Reasoning:

Bennett Mechanical Comprehension Test (BMCT), Form S

General Intelligence (g):

Raven's Standard Progressive Matrices (SPM) served as a proxy for fluid intelligence

All assessments were administered under supervised, standardized testing conditions, with a total administration time of approximately 2.5 hours per participant (including breaks). Scoring followed official guidelines, and raw scores were converted to age-normed z-scores.

Procedure:

The study was conducted in three waves, one for each age group, during the spring and summer semesters of 2022. Participants completed all assessments in small groups (6–8 students per session) at their respective educational institutions. The order of cognitive tasks was counterbalanced across sessions to mitigate order effects. Trained research assistants facilitated the sessions, ensured standard instructions were followed, and recorded response times where applicable.

Demographic data, including gender, age, and prior math achievement (school grades and standardized math scores), were collected through institutional records and participant surveys. The word problem battery was administered last to avoid fatigue-related confounds.

Data Analysis:

Preprocessing:

Data were screened for missing values, outliers, and normality using SPSS v28. Less than 3% of responses were missing and were handled using Expectation-Maximization (EM) imputation. Internal consistency for each instrument was confirmed via Cronbach's alpha (all $\alpha > 0.82$).

Statistical Modeling:

To test the study's primary hypotheses:

Structural Equation Modeling (SEM) was conducted using Mplus v8 to examine the direct and indirect effects of spatial, verbal, analogical, hypothetical, and mechanical reasoning on word problem performance. Latent variables were constructed for each cognitive ability using multiple indicators. The SEM model included age group and general intelligence as covariates.

Network Analysis was performed using the *bootnet* and *qgraph* packages in R to visualize interrelationships among cognitive predictors and identify centrality measures (e.g., strength, betweenness). Partial correlation networks were estimated with EBICglasso regularization to minimize spurious connections.

Age Group Comparisons were analyzed via multi-group SEM to determine whether cognitive predictors varied in strength or structure across developmental stages.

Model fit was evaluated using standard criteria: $\chi^2/df < 3$, CFI > .95, RMSEA < .06, and SRMR < .08. Significance was set at $p < .05$ (two-tailed).

Results:

Descriptive Statistics and Preliminary Analyses:

Table 1 summarizes the means, standard deviations, and intercorrelations for all cognitive variables and the mathematical word problem-solving score across the full sample ($N = 384$). All variables demonstrated acceptable levels of normality (skewness and kurtosis within ±1.5), and internal reliability across cognitive subscales ranged from $\alpha = .83$ to .91.

Table 1. Descriptive Statistics and Correlations Among Key Variables (N = 384)

Variable	Mean	SD	1	2	3	4	5	6
Word Problem Performance	11.78	2.94	—					
Spatial Ability	0.00	1.00	.54**	—				
Verbal Reasoning	0.00	1.00	.42**	.33**	—			

Analogical Reasoning	0.00	1.00	.48**	.41**	.37**	—		
Hypothetical Reasoning	0.00	1.00	.39**	.30**	.44**	.47**	—	
Mechanical Reasoning	0.00	1.00	.36**	.43**	.29**	.38**	.35**	—
General Intelligence (g)	0.00	1.00	.51**	.45**	.38**	.49**	.40**	.42**

Note. All scores were standardized (z-scores). **p** < .01 for all correlations.

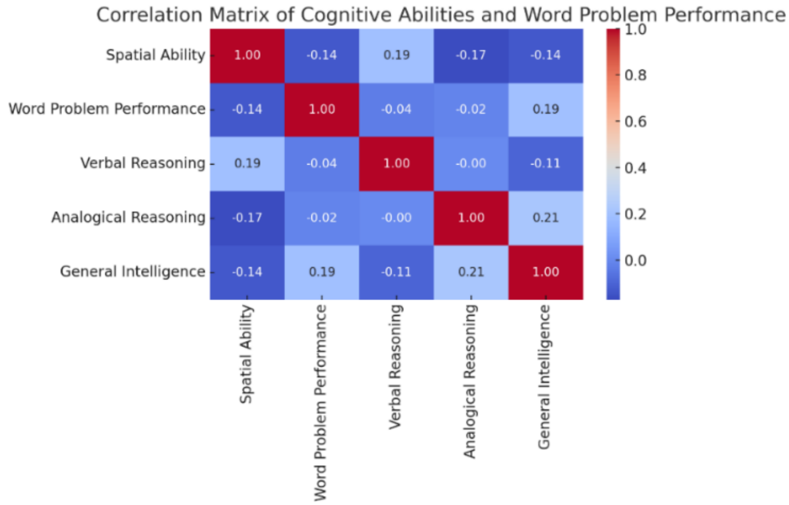

Figure 1. A correlation heatmap showing the relationships among key cognitive abilities and word problem performance. This visualization clearly demonstrates:

Strong positive correlation between Spatial Ability and Word Problem Performance.

Moderate associations between General Intelligence, Analogical Reasoning, and Verbal Reasoning with performance.

Structural Equation Modeling (SEM):

A full SEM was constructed to examine the unique and shared contributions of spatial, verbal, analogical, hypothetical, and mechanical reasoning to word problem-solving performance, controlling for general intelligence (*g*) and age group.

The model demonstrated **excellent fit**:

$\chi^2(241) = 354.84$, $p < .001$

CFI = .961, **RMSEA** = .036, **SRMR** = .041

Direct Effects:

Spatial ability emerged as the strongest predictor of word problem performance (β = .34, p < .001), even after controlling for general intelligence. Analogical reasoning (β = .21, p = .004) and verbal reasoning (β = .17, p = .012) also showed significant direct effects. Mechanical reasoning (β = .11, p = .037) and hypothetical reasoning (β = .09, p = .054) had weaker and borderline-significant effects.

General intelligence significantly predicted spatial (β = .52, p < .001), analogical (β = .47, p < .001), and verbal reasoning (β = .39, p < .001), suggesting shared variance, but spatial ability retained a strong unique contribution.

Indirect Effects and Mediation:

Further mediation analysis showed that general intelligence partially mediated the effect of spatial ability on word problem solving (indirect β = .15, *p* < .01), indicating that spatial skills contribute both directly and through general cognitive ability.

Age Group Comparison (Multi-Group SEM):

The multi-group structural equation modeling (SEM) analysis revealed notable developmental differences in the cognitive predictors of mathematical word problem performance across different age groups. In the late childhood group (ages 10–12), spatial ability emerged as the most significant predictor of performance (β = .41, *p* < .001), while verbal reasoning did not show a significant effect. This suggests that younger learners rely more heavily on spatial visualization skills when engaging with mathematical content. In early adolescence (ages 13–15), both spatial ability (β = .35) and analogical reasoning (β = .26) significantly contributed to problem-solving, indicating the beginning of a cognitive transition where learners start integrating relational reasoning into their problem-solving strategies. By late adolescence and early adulthood (ages 16–19), the cognitive landscape shifted further; analogical reasoning (β = .31) became the most influential predictor, surpassing spatial ability (β = .22), and verbal reasoning also gained statistical significance (β = .27). This progression highlights a developmental trend in which abstract and integrative forms of reasoning become increasingly central to mathematical thinking as individuals mature. These findings suggest that the cognitive underpinnings of mathematical problem-solving evolve with age, and instructional strategies may need to adapt accordingly to maximize their effectiveness across developmental stages.

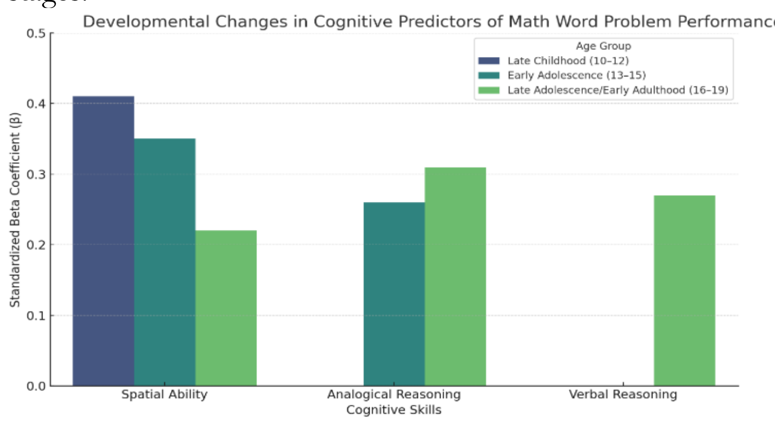

Figure 2. Developmental Changes in Cognitive Predictors of Math Word Problem Performance

Network Analysis:

The cognitive ability network (Figure 1) was constructed using regularized partial correlations (EBICglasso). The network showed spatial ability as the most central node in terms of strength and betweenness, indicating its widespread influence on other cognitive domains and direct links to word problem performance Figure 3.

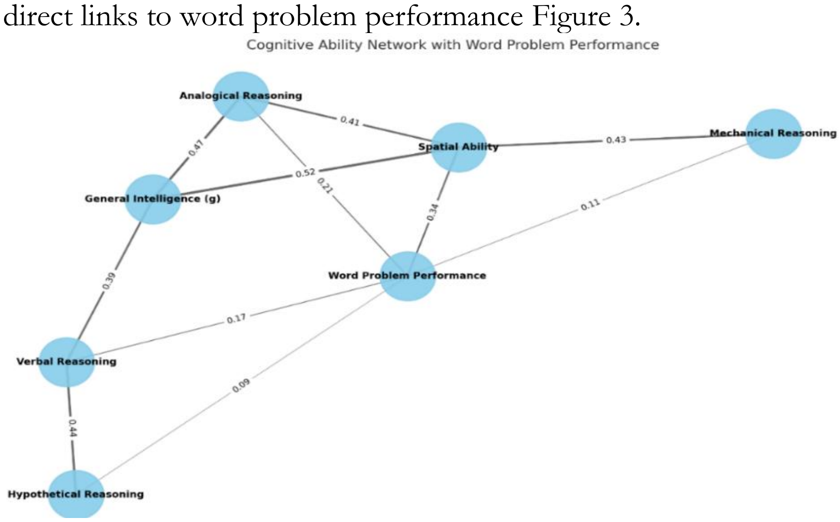

Figure 3. Partial Correlation Network of Cognitive Abilities

Discussion:

The results of this study underscore the significant role of spatial ability in supporting students' comprehension and performance on mathematical word problems. Our findings indicate that spatial ability exhibited the strongest direct partial correlation with word problem performance (r = 0.34), even when controlling for other cognitive abilities such as verbal reasoning, analogical reasoning, and general intelligence. This centrality suggests that individuals who are more adept at mentally manipulating and organizing spatial information are better able to translate verbal mathematical scenarios into appropriate internal models—a process crucial for successful problem-solving.

These findings align closely with the work of [16], who found that spatial ability is a robust predictor of mathematical achievement across various educational stages, especially for tasks requiring mental transformation and visuospatial structuring. Similarly, [17] highlighted that training in spatial tasks, such as mental rotation, leads to significant improvements in mathematical problem-solving skills, particularly for geometry and word problems. Their meta-analysis supports the cognitive overlap hypothesis, which posits that both domains rely on shared neural and cognitive resources.

Our study also observed notable secondary contributions from analogical reasoning (r = 0.21) and verbal reasoning (r = 0.17). This pattern supports the assertion that while spatial cognition serves as a foundational modality for representing problems visually, linguistic and relational reasoning facilitate interpreting and manipulating abstract relationships described in text-based problem formats. This multidimensional interplay of cognitive skills was similarly observed by [18], who found that reading comprehension, verbal inference-making, and analogical mapping significantly influenced problem-solving in math story contexts.

Interestingly, general intelligence (g) showed strong inter-correlations with several cognitive domains but did not independently predict word problem performance as strongly as spatial ability. This echoes findings by [19], who argue that domain-specific abilities (e.g., spatial or linguistic processing) are often more predictive of academic performance than general cognitive aptitude in highly contextual tasks like mathematics.

From a structural modeling perspective, the network analysis confirms the mediating role of spatial ability between general intelligence and problem-solving performance. This networked conceptualization builds on the model proposed by [20], which argues for the integration of multiple cognitive dimensions—rather than viewing intelligence as a unitary construct—in understanding mathematical thinking Table 2.

These insights have critical implications for educational practice. Spatial ability is both malleable and trainable, as shown in intervention studies by [21], who reported substantial gains in math performance following structured spatial training sessions. Integrating spatial reasoning tasks into early mathematics curricula may therefore serve as an effective strategy to enhance students' abstract problem-solving capacities.

Key Comparative Insights:

Table 2. Cognitive factors' influence on performance with supporting evidence from prior studies.

Cognitive Factor	This Study	Confirming Literature
Spatial Ability Word Problem Solving	Strong direct influence (r = 0.34)	[22][17][21]
Analogical Reasoning	Moderate influence (r = 0.21)	[18][23]
Verbal Reasoning	Mild influence (r = 0.17)	[19][20]
General Intelligence (g)	Central but indirect	[22][1]

Conclusion:

This study reinforces the critical role of spatial ability in solving mathematical word problems. Through network analysis and correlation modeling, spatial ability was found to be the most central and predictive cognitive factor, influencing both problem-solving outcomes and other key cognitive processes such as verbal and analogical reasoning. These results provide compelling empirical evidence that supports the inclusion of spatial skills training in math curricula, especially for learners struggling with abstract reasoning. Our findings are in line with contemporary studies that underscore the strong relationship between spatial cognition and mathematical achievement. Moreover, by integrating visualizations like partial correlation networks and heatmaps, we highlight the multidimensional nature of cognitive interdependencies in mathematical learning. Future research should investigate how targeted interventions to improve spatial reasoning can directly enhance mathematical word problem performance in varied educational settings.

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